Web problems

Magnetism

Problems labelled A are straightforward, those marked B are supposed to be more demanding, those marked C are intended to make students think, and those marked S are synoptic. There are online physics tutorials on WebLearn under Physics for Chemists.

1. Electric current.

Drift speed

1.1A A wire of cross-sectional area A contains n conduction electrons per unit volume. Show that the electrical current in the wire is equal to $i = nAve$ where e is the charge on the electron and v is the drift speed.

1.2A Early electricity meters for dc supply diverted 11% of the current supplied into an electrolysis cell where zinc ions were reduced to zinc. The mass of zinc deposited was then used to measure the charge supplied to the house. If during a month 65.4 g of zinc was deposited, how much charge was supplied?

1.3A A silver wire of radius 800 μm carries a current of 15 mA. Assuming that each silver atom releases one conduction electron calculate the drift speed of the electrons in this wire. The molar volume of silver is 10.27 $cm^3\ mol^{-1}$.

1.4A Silver iodide is a fast ion conductor. Above 420 K the silver ions become mobile and conduct electricity, the iodide ions remain fixed. A disk of silver iodide of radius 1.0 cm carries a current of 30 mA. Calculate the drift speed of the silver ions. The density of silver iodide is 5683 kg m^{-3} and the relative formula mass is 234.773.

1.5A In a conductivity cell of cross-sectional area 1.00 cm^2 containing a 1.00 mM solution of RbBr a current of 1.56 μA flows. Assuming that the drift velocities of the two ions are equal and opposite find them.

Resistivity and conductivity.

1.6A A 100 W light bulb operating at 220 V passes a current of 0.417 A. Calculate the resistance of the bulb.

1.7A The resistivity of a copper wire is $1.72\times10^{-8}\ \Omega$ m. If the diameter of the wire is 2.05 mm calculate the resistance of a 30 m length of wire.

1.8A A disc of α-AgI of radius 1.0 cm and thickness 2.0 mm is found to have a resistance of 4.1 Ω. Calculate its resistivity and its conductivity.

Molar conductivity

1.9B A conductivity cell is filled with a standard solution of aqueous KCl of concentration 0.100 mol dm^{-3} and found to have a resistance of 28.44 Ω. The molar conductivity of KCl is 129 S cm^2 mol^{-1}. In the same cell a solution of an unknown concentration of NaOH is found to have a resistance of 31.60 Ω. Calculate the concentration of this solution. The molar conductivity of NaOH is 247.8 S cm^2 mol^{-1}.

1.10B 50 cm^3 of a 10.0 mmol dm^{-3} solution of NaOH was titrated with a 1.00 mol dm^{-3} solution of HCl. Calculate the conductivity κ of the resulting solution
(i) at the start of the titration
(ii) at the end point of the titration
(iii) when twice as much HCl has been added as at the end point.
Sketch a graph of the concentrations of each species and a graph of the conductivity as a function of the volume of acid added and explain the form of the graph.
[Molar ionic conductivities at infinite dilution: Na^+ = 50.15, H^+ = 349.85, Cl^- = 76.35, OH^- = 197.6 S cm^2 mol^{-1}.]

1.11B The following molar conductivities are tabulated at infinite dilution:

NaOAc	91.04 S cm^2 mol^{-1}
NaCl	126.5 S cm^2 mol^{-1}
HCl	426.2 S cm^2 mol^{-1}

(*a*) Calculate the molar conductivity of HOAc at infinite dilution.

(*b*) How would you expect the molar conductivity of HOAc to vary with concentration?

1.12A The following molar conductivities are tabulated at infinite dilution:

Na^+	50.15 S cm^2 mol^{-1}
Cl^-	76.35 S cm^2 mol^{-1}

(*a*) Calculate the molar conductivity of NaCl at infinite dilution.
(*b*) Calculate the conductivity of a 0.032 M solution of NaCl.
(*c*) Calculate the mobilities of Na^+ and Cl^- ions.
(*d*) A potential difference of 1.28 V is applied across two plates separated by 1.00 cm in the solution of (*b*). Find (i) the current density and (ii) the drift speed of each ion.

2. Magnetic force

Force on a moving charged particle in a magnetic field

2.1A An ion of charge +1*e* and mass 7.016 u travelling in the +*x* direction at a speed of 450 m s^{-1} enters a magnetic field of 1.25 T directed in the +*z* direction.
(i) calculate the force on the ion when it enters the field
(ii) calculate the radius of the circular motion it undergoes.

2.2B In a mass spectrometer an ion with a charge +1*e* is accelerated through a potential difference of 10.0 kV and then passes into a constant magnetic field of 0.300 T perpendicular to the path of the ion. It undergoes a circular motion of radius 42.4 cm. Deduce the relative ionic mass.

2.3C In a TV tube electrons are accelerated through a voltage of 8.00 kV and are then incident on a fluorescent screen 0.40 m away. There is a stray vertical magnetic field from an nmr machine one floor above. How strong must this field be to divert the electrons 1 mm from their intended targets on the screen? In a colour TV tube this effect can cause electrons to miss the correct colour mask and lead to colour distortions. Compare this field with the earth's magnetic field in Oxford, 47 μT.

Crossed fields

2.4 The charge/mass ratio of the electron may be measured as follows, based on the experiments of JJ Thomson in the 1890s. Electrons are accelerated through a pd of 2.00 kV. They then enter a region of perpendicular crossed electric and magnetic fields. If the deflecting electric field is 10.0 kV cm^{-1} a magnetic field of 37.7 mT is required for the beam to be undeflected. Calculate the charge/mass ratio of the electron and compare it with the accepted value.

2.5 In a mass spectrometer it is required to select ions of mass 254 u. Ions are accelerated through a potential difference of 10.0 kV and the deflection electric field is 10.0 kV m^{-1}, what magnetic field is required.

2.6 In a gold fim of thickness 1 μm a current of 1 mA produces a Hall voltage of 31.8 nV measured from side to side of the film, in a perpendicular magnetic field of 0.3 T, estimate the number density of current carriers in gold. You may assume that the film has a rectangular cross-section.

2.7 $RbAg_4I_5$ is a fast ion conductor.

(*a*) Its crystal structure is cubic with a unit cell of side 11.24 Å which contains 4 formula units. Find the number density for each of Rb^+, Ag^+ and I^- ions in the solid.

(*b*) A cuboidal sample whose thickness in the *y* direction is 1.0 mm carries a current of 0.35 A in the +*x* direction. A magnetic field of 1.0 T is applied in the +*y* direction generates a Hall potential difference of 200 nV in the *z* direction, with the upper face positive.

(i) Deduce the sign of the charge on the current carriers.

(ii) Calculate the number density of the current carriers and hence deduce the likely nature of the mobile ions.

Magnetic field induced by current

3.1A A circular loop of wire of radius 0.60 m carries a current of 3.00 A. Calculate the magnetic field at the centre of the loop.

Magnetic dipoles

3.2A Find an expression for the magnetic moment of an electron in the 1st Bohr orbit and evaluate it (you may neglect spin).

3.3A The magnetic moment of a nuclear spin 1/2 nucleus has the magnitude $\frac{\sqrt{3}}{2} g\mu_N$, where *g* is a numerical factor which depends on the structure of the nucleus and μ_N is a fundamental constant called the nuclear magneton and equals 5.051×10^{-27} J T^{-1}. When a proton (*g* = 5.586) is introduced into a magnetic field the quantisation of angular momentum ensures that the component of the magnetic moment in the direction of the field may only take the values $\mu_z = \pm\frac{1}{2} g\mu_N$. Calculate the energy of the proton in these two states if the field strength is 9.396 T. Hence calculate the frequency of the transition from one orientation to the other.

Current induced by magnetic field.

3.3B Explain why two protons in different chemical environments have different chemical shifts in the NMR spectrum.

3.4S In the Bohr model of a 1 electron atom (e.g. H, He^+, Li^{2+} etc.) an electron executes a circular motion around the nucleus. The centripetal force of the circular motion is equal to the electrostatic attractive force between the nucleus and the electron. A second relationship between speed and radius is given by Bohr's hypothesis that angular momentum is quantised in integer units of $\hbar$ results in energy levels.
Assuming that the nucleus has a charge *Ze*,
(*a*) Find expressions for the radius of the orbit and the tangential speed of the electron in terms of fundamental constants, *Z* and the quantum number *n*.
(*b*) Deduce an expression for the energy of the electron and evaluate the energy difference between the states *n* = 1 and 2.
(*c*) Deduce an expression for the magnetic field at the nucleus due to the

circulation of the electron.

(*d*) In the frame of reference of the electron the nucleus executes a circular motion with the electron at the centre. Deduce an expression for the magnetic field the electron experiences as a result of this motion. Evaluate this field for the H atom in the second Bohr orbit (n = 2).

(*e*) An electron also has a spin magnetic moment. In a magnetic field B the interaction energy of the electron spin magnetic moment with the field can only take the values $\pm\mu_B B$ (see quantum lectures). Use the result of (*d*) to find an expression for the energy difference between the two different spin states of the electron due to its interaction with field generated by its own orbital motion in Bohr orbit n. Calculate this energy difference for n = 2 and compare it to the experimental value 7.27×10^{-24} J.

(*f*) A more detailed analysis gives the familiar quantum numbers n, l and ml. To a first approximation (Schrödinger) the energy of electron depends only on the principal quantum number,

$E_n = -\dfrac{Z^2 m_e e^4}{2n^2\hbar^2(4\pi\varepsilon_0)^2}$. Compare this result with your result in (*b*).

(*g*) A higher order of theory [Dirac, the details are sadly beyond the scope of a chemistry degree] predicts that whenever the orbital quantum number is non-zero, the energy level is split into two by the magnetic coupling of the spin and orbital angular momentum and that the splitting is given by

$\Delta E_{nl} = \dfrac{\mu_0}{4\pi}\dfrac{m_e e^8}{\hbar^4(4\pi\varepsilon_0)^3}\dfrac{Z^4}{2n^3 l(l+1)}$. Compare this with the formula you obtained from the Bohr model for this coupling in (*d*) and comment on the agreement of the numerical values found in (*d*).

4. Electromagnetic waves

Frequency, wavelength and speed.

4.1A A wave is represented by $\psi_1 = A\cos(kx + \omega t)$ where $k = 2.47\times10^{7}$ m^{-1} and ω = 7.41×10^{15} s^{-1}. Show that this represents a travelling wave and deduce its wavelength, frequency, speed and direction of travel.

4.2A A medical ultrasound scanner has a frequency of 200 MHz. Ultrasound propagates through the human body with a speed of 1500 m s^{-1}. What will be the wavelength of the ultrasound?

Law of refraction, refractive index and speed of light.

4.3A The speed of light of wavelength 656 nm in ethanol is 2.204×10^{8} m s^{-1}.
(*a*) Calculate the refractive index of ethanol.
(*b*) Calculate the wavelength of this light when it emerges into a vacuum?

4.4A Light of a certain frequency has a wavelength of 532 nm in water (n = 1.333). What is the wavelength in carbon disulphide (n = 1.628)?

4.5A The wavelength of one of the 3p – 2s transitions in the H atom is measured in air to be 656.27247 nm. However, for the purposes of tabulating energy levels it is necessary to work out the corresponding wavelength in vacuum. Calculate this quantity using the appropriate refractive index of air, 1.0002762.

4.6B Explain why a straight stick appears to be bent when it is dipped into water.

4.7B A point source of light lies below the surface of a lake.
(*a*) Find the critical angle for total internal reflection.
(*b*) If the source is 0.320 m below the surface of the lake what is the diameter of the circle on the surface of the lake through which light can emerge? Take the refractive indices of water and air to be 1.333 and 1.000 respectively.

4.8C Show how Snell's law of refraction is a consequence of Fermat's principle of least time.

Principle of superposition, interference.

4.9B A wave is represented by $\psi_1 = A\cos(kx - \omega t)$, ψ_1 interferes with a second wave ψ_2 = $\psi_1 = A\cos(kx - \omega t + \pi/3)$. Find the amplitude and phase of the resultant wave.

4.10A When you are tuning a stringed instrument you can tell if a string is slightly out of tune by playing the same note simultaneously on two strings. If the strings are not quite in tune you hear the two notes 'beating' against each other. Explain this phenomenon by considering the superposition of two cosine waves with almost the same frequency. You only need consider the time-dependence of the wave. Sketch the resulting function. [This type of phenomenon is very important in many applications - AM radio, Fourier transform spectroscopy and mode-locking lasers, for example.]

4.12B A Young's slits experiment is set up in which two narrow slits of separation *d* are illuminated with light of wavelength λ. The diffraction pattern is viewed on a screen at a long distance *D*.

(*a*) Derive an expression for the angles at which bright interference fringes will be observed, and for the distance of these fringes from the centre of the pattern on the screen.

(*b*) If *D* = 1 m, λ = 600 nm and the distance from the centre of the fringe pattern to the 10th bright band is 30 mm, calculate the distance between the two slits.

4.13B Consider two travelling waves $\psi_1 = A\cos(kx - \omega t)$ and $\psi_2 = A\cos(kx + \omega t)$, propagating in opposite directions. Calculate the wave resulting from this interference.